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## Gauge dependence in QED amplitudes in expanding de Sitter space

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We consider first order transition amplitudes in external fields in QED in the expanding de Sitter space and point out that they are gauge dependent quantities. We examine the gauge variations of the amplitudes assuming a decoupling of the interaction at large times, which allows to conclude that the source of the problem lies in the fact that the frequencies of the modes in the infinite future become independent of the comoving momenta. We show that a possibility to assure the gauge invariance of the external field amplitudes is to restrict to potentials which vanish sufficiently fast at infinite times, and briefly discuss a number of options in the face of the possible gauge invariance violation in the full interacting theory.

*Keywords:* de Sitter space; quantum electrodynamics; gauge dependence.

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### 1. Introduction

de Sitter (dS) space has received a special attention in the study of quantum field effects in curved spacetimes. It plays a fundamental role in inflationary cosmology models, and due to its high symmetry it provides a convenient laboratory for obtaining closed form results. dS space is also of special interest due to the fact that it allows a significant number of effects which have no counterpart in the Minkowski space. For example, a well known result is that for the minimally coupled free massless scalar field there is no dS-invariant vacuum state, and that in certain vacua the quantum fluctuations can become arbitrarily large.<sup>1,2</sup> Other notable examples are that free particles can decay in multiple copies of themselves,<sup>3–7</sup> the interacting vacuum could be highly unstable and lead to a massive particle production,<sup>8–15</sup> or the spacetime itself could be instable due to quantum gravitational effects.<sup>16–19</sup>

While the effects above have nothing intrinsically problematic, it also seems that dS space involves a series of less trivial features, which indicate a departure from the flat space theory at a more fundamental level: there are opinions that a well-defined  $S$ -matrix may not exist,<sup>25–27</sup> infrared divergences in Feynman diagrams systematically show up,<sup>20–24</sup> or unitarity might be violated by interacting fields.<sup>28,29</sup>

The main intention of the present paper is to point out another fact which seems to put the theory in dS space on an unequal footing with that in flat space. We will focus our attention on first order transition amplitudes in external fields in spinor QED in the expanding patch of dS space. We will show that these amplitudes are gauge dependent quantities. Despite the rather large amount of work on QED effects in dS space (see below), it appears that this fact has passed unnoticed. Here, we will be content to make clear that this is indeed the case, and briefly suggest a number of possibilities to deal with this situation.

Gauge invariance in QED calculations typically refers to the invariance of the  $S$ -matrix elements under gauge transformations in (1) the wave functions of the external photon lines, (2) the external potential, and (3) the photon propagator. Amplitudes at tree level for various QED processes in dS space involving (1) and (2) in a particular gauge have been explicitly obtained in Refs. 31–40. Tree-level QED amplitudes in a flat FRW universe, which most probably can be generalized to a dS space, can be also found in Refs. 41–55. Loop calculations involving (3) with the photon propagator in a dS invariant form in the Feynman gauge have been done in Ref. 56, in the Landau gauge in Refs. 57–60, and using a dS non-invariant analogue of the Feynman gauge<sup>61</sup> in Refs. 56, 57, 62–66. One should stress that the evaluation of diagrams and implicitly the check of gauge invariance at higher orders is generally significantly more difficult than in flat space because of the lack of the machinery of Fourier transforms, and also due to the more complex form of the propagators.<sup>a</sup>

To our knowledge, the only systematic comparison between calculations in different gauges in dS space was made in Ref. 57, where the one-loop self-mass operator of a charged scalar field was obtained both in the Landau<sup>68</sup> and in the dS non-invariant gauge.<sup>61</sup> The self-masses in the two gauges differ, but this is nevertheless not enough to conclude the violation of gauge invariance, which refers to physically measurable (on-shell) quantities. Such a quantity can be obtained by considering the one-loop corrected evolution of the scalar field.<sup>57</sup> It turns out that for an initial plane wave the evolution in the two gauges leads to a qualitatively different time dependence at late times, which can be seen as a gauge dependence of the results. Another significant discrepancy was noted in Ref. 56, where it was found that the self-mass in the Feynman gauge contains extra on-shell singularities compared to that in the Landau gauge.

Our calculation will be considerably simpler, as we will focus on tree-level amplitudes. The plan is as follows. We will consider transition amplitudes between the vacuum and a fermion-antifermion state in an external potential  $A_\mu$ , with the particles in both the *in* and *out* states defined by the Bunch-Davies modes. Similar amplitudes were used to discuss the possible unitarity violation<sup>27, 28</sup> and the vac-

<sup>a</sup>The photon propagator in a dS invariant form in an arbitrary  $R_\lambda$  gauge was only recently obtained in Ref. 71. The propagator of Allen and Jacobson<sup>67</sup> corresponds to the Feynman gauge  $\lambda = 1$ . The Landau gauge  $\lambda \rightarrow \infty$  is discussed in Ref. 68. See also Refs. 69, 70 for an arbitrary gauge.

uum decay<sup>27,29,30</sup> in the expanding dS space. In particle production calculations, one usually introduces a different set of *out* modes in order to define the physical particles at late times, but as we will point out this is irrelevant to our question. It will be then an easy task to show that for potentials of the form  $A_\mu = \text{constant}$  these amplitudes do not vanish. Since such potentials correspond to a pure gauge, the result is incompatible with the gauge invariance of the theory.<sup>b</sup>

We will make a further step and identify the mechanism which lies behind the gauge dependent amplitudes. Introducing a decoupling of the interaction at infinite times, one can rewrite the gauge variations of the amplitudes as an integral which contains the time derivative of the decoupling factor. In the Minkowski space, the purely oscillatory time dependence of the modes assures that this quantity always vanishes for an adiabatic decoupling. As we will see, this does not happen in dS space. We will explain that the cause lies (not surprisingly) in the infinite expansion of space in the infinite future, which makes all the frequencies of the modes approach the same value at late times. The situation is somehow analogous to that of a transition with a vanishing Bohr frequency, which can formally lead to divergent amplitudes. In our case, the pathology manifests in the fact that the gauge variations of the amplitudes do not vanish for an adiabatic decoupling.

We will also propose a simple solution for assuring the gauge invariance of the external field amplitudes. This essentially consists in restricting the calculations to external potentials which vanish sufficiently fast at infinite times, in which conditions the gauge variations of the amplitudes are rigorously zero. It remains to be seen with more calculations in specific cases if our proposal is a satisfactory solution to the problem.

The paper is organized as follows. In Sec. 2 we establish the general form of the amplitudes. In Sec. 3 we exhibit the amplitudes which break the gauge invariance, and in Sec. 4 we identify the mechanism which leads to such quantities. In Sec. 5 we discuss the condition for restoring the gauge invariance of the amplitudes. We end in Sec. 6 by briefly listing a number of options to deal with the possible gauge dependence in the fully quantized theory.

## 2. The transition amplitudes

The line element of the expanding dS space is

$$ds^2 = dt^2 - e^{2Ht} d\mathbf{x}^2, \quad (1)$$

where  $H > 0$  is the expansion parameter. It is convenient to define the conformal time

$$\eta = -\frac{1}{H} e^{-Ht}, \quad \eta \in (-\infty, 0), \quad (2)$$

<sup>b</sup>We have noted this fact for the analogous amplitudes in scalar QED in a different context in Ref. 77.

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in terms of which the metric reads

$$ds^2 = \frac{1}{(H\eta)^2} (d\eta^2 - d\mathbf{x}^2). \quad (3)$$

As is well known, in order to deal with spinor fields in curved spacetimes it is necessary to introduce a local orthonormal frame  $\{e_{\hat{\alpha}}\}$ . We choose it to be the Cartesian frame defined by (in conformal coordinates)

$$e_{\hat{\alpha}}^{\mu} = -(H\eta) \delta_{\alpha}^{\mu}. \quad (4)$$

The first order QED amplitudes in the external potential  $A_{\mu}$  have the general form

$$\mathcal{A}_{i \rightarrow f} = -ie \int d^4x \sqrt{-g} \bar{\psi}_f \gamma^{\alpha} \psi_i A_{\hat{\alpha}}, \quad A_{\hat{\alpha}} = e_{\hat{\alpha}}^{\mu} A_{\mu}, \quad (5)$$

where all notation is conventional. We will focus on the case when the initial and final wave functions are solutions of the Dirac equation of a definite momentum  $\mathbf{p}$  and helicity  $\lambda = \pm \frac{1}{2}$ . It is useful to introduce

$$k = \frac{m}{H}, \quad \nu_{\pm} = \frac{1}{2} \pm i \frac{m}{H}, \quad (6)$$

where  $m$  is the mass of the Dirac field. We also need the unit norm helicity two-spinors  $\xi_{\lambda}$  and  $\eta_{\lambda}$  defined by

$$\frac{1}{2} (\mathbf{n}_p \cdot \boldsymbol{\sigma}) \xi_{\lambda}(\mathbf{p}) = \lambda \xi_{\lambda}(\mathbf{p}), \quad \eta_{\lambda}(\mathbf{p}) = i\sigma_2 \xi_{\lambda}^*(\mathbf{p}), \quad \mathbf{n}_p = \frac{\mathbf{p}}{p}. \quad (7)$$

The solutions of the Dirac equation mentioned above can then be written as<sup>72-74</sup> ( $\sigma \equiv 2\lambda$ ):

$$u_{\mathbf{p}, \lambda}(\eta, \mathbf{x}) = \frac{\sqrt{\pi p/H}}{2(2\pi)^{3/2}} \times (H\eta)^2 \begin{pmatrix} e^{+\frac{\pi k}{2}} H_{\nu_-}^{(1)}(-p\eta) \xi_{\lambda}(\mathbf{p}) \\ \sigma e^{-\frac{\pi k}{2}} H_{\nu_+}^{(1)}(-p\eta) \xi_{\lambda}(\mathbf{p}) \end{pmatrix} e^{i\mathbf{p}\mathbf{x}}, \quad (8)$$

and

$$v_{\mathbf{p}, \lambda}(\eta, \mathbf{x}) = \frac{\sqrt{\pi p/H}}{2(2\pi)^{3/2}} \times (H\eta)^2 \begin{pmatrix} -\sigma e^{-\frac{\pi k}{2}} H_{\nu_-}^{(2)}(-p\eta) \eta_{\lambda}(\mathbf{p}) \\ e^{+\frac{\pi k}{2}} H_{\nu_+}^{(2)}(-p\eta) \eta_{\lambda}(\mathbf{p}) \end{pmatrix} e^{-i\mathbf{p}\mathbf{x}}, \quad (9)$$

where  $H_{\nu}^{(1,2)}(z)$  are the Hankel functions of the first and second kind. The four-spinors are in the standard Dirac representation with the matrix  $\gamma^0$  diagonal. The solutions (8) and (9) can be identified as positive and negative frequency modes, respectively, in the sense that in the infinite past they are purely oscillatory in the conformal time, i.e.<sup>c</sup>

$$u \sim e^{-ip\eta}, \quad v \sim e^{ip\eta}, \quad \eta \rightarrow -\infty. \quad (10)$$

<sup>c</sup>The vacuum defined by these modes is the Bunch-Davies vacuum.<sup>75</sup>

It is convenient for our discussion to consider amplitudes for particle production from the initial vacuum. The wave functions  $\psi_f$  and  $\psi_i$  have then to be identified with  $u_{\mathbf{p},\lambda}$  and  $v_{\mathbf{p}',\lambda'}$ , in which conditions

$$\mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'} = -ie \int d^4x \sqrt{-g} \bar{u}_{\mathbf{p},\lambda} \gamma^\alpha v_{\mathbf{p}',\lambda'} A_{\hat{\alpha}}. \quad (11)$$

It will also be sufficient to restrict to potentials  $A_\mu$  of the following form:

$$A_0 = 0, \quad A_i = A_i(\eta), \quad i = 1, 2, 3. \quad (12)$$

Let us write Eq. (12) in a more explicit way. Notice that thanks to the factorizable dependence on  $\eta$  and  $\mathbf{x}$  in Eqs. (3), (4), (8), (9) and (12) the temporal and spatial integrations can be separately performed. The integration with respect to  $\mathbf{x}$  is immediate and leads to a delta-Dirac function. A simple calculation then shows that the remaining factors can be organized as follows ( $\sigma^i$  are the Pauli matrices):

$$\mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'} = -ie \delta^3(\mathbf{p} + \mathbf{p}') [\xi_\lambda^+(\mathbf{p}) \sigma^i \eta_{\lambda'}(\mathbf{p}')] F_i(p)_{\sigma\sigma'}, \quad (13)$$

where we collected the integrals with respect to  $\eta$  in

$$F_i(p)_{\sigma\sigma'} = f_i^+(p) - \sigma\sigma' f_i^-(p), \quad (14)$$

$$f_i^\pm(p) = \frac{\pi p}{4} e^{\pm k\pi} \int_{-\infty}^0 d\eta \eta A_i(\eta) [H_{\nu_\pm}^{(2)}(-p\eta)]^2. \quad (15)$$

It is clear that the specific ‘dS part’ of the amplitudes is encoded in the  $\eta$ -integrals (15). One can easily check that in the flat space limit  $H \rightarrow 0$  the amplitudes (15) reduce, as expected, to their analogues in the Minkowski space.<sup>d</sup>

### 3. Gauge dependence of the amplitudes

We now show that the amplitudes (13) are not gauge invariant. Let us choose an external potential of the form

$$A_0 = 0, \quad A_i = \text{constant}. \quad (16)$$

The corresponding field strength  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is identically zero, so that the potential is a pure gauge. Gauge invariance in these conditions requires the amplitudes to vanish.

Let us look more closely at Eq. (13) in this case. Since the directions of  $\mathbf{p}$  and  $A_i$  are arbitrary, the vanishing of  $\mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'}$  for all helicities  $\lambda, \lambda'$  requires  $F_i(\mathbf{p})_{\sigma\sigma'}$  to vanish. The arbitrariness of  $\sigma, \sigma'$  further implies that  $f_i^\pm(\mathbf{p})$  must vanish. For the potentials (16) these functions are

$$f_i^\pm(p) \sim \int_{-\infty}^0 d\eta \eta [H_{\nu_\pm}^{(2)}(-p\eta)]^2. \quad (17)$$

<sup>d</sup>The limits for the Hankel functions with  $\nu_\pm = \frac{1}{2} \pm ik$  can be found e.g. in Eq. (5.6) of Ref. 3.

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It is clear that these integrals are not identically zero. This means that the amplitudes cannot be gauge invariant.

For safety, let us make sure that the integrals (17) are not ill-defined. The behavior of the functions  $H_\nu^{(2)}(z)$  for small and large arguments  $z$  is

$$H_\nu^{(2)}(z) \simeq -\frac{i}{\pi} \Gamma(\nu) \left(\frac{z}{2}\right)^{-\nu}, \quad z \rightarrow 0, \quad H_\nu^{(2)}(z) \simeq \sqrt{\frac{2}{\pi z}} e^{-i(z-\frac{\pi}{2}\nu-\frac{\pi}{4})}, \quad z \rightarrow \infty. \quad (18)$$

The first relation shows that for  $\eta \rightarrow 0$  the integrand is  $\sim \eta^{\pm 2ik}$ , which puts no convergence problems. The second one implies that for  $\eta \rightarrow -\infty$  the integrand is a pure phase  $\sim e^{2ip\eta}$ , which is also not problematic (the integral can be made well-defined by introducing a small convergence factor, corresponding to a decoupling of the interaction in the far past). For example, a simple way to see that  $f_i^\pm(p)$  are not identically zero is by considering  $p \rightarrow \infty$ . Using the second relation in Eq. (18) the integration is immediate and one finds  $f_i^\pm(p) \sim p^{-1}$ .

It is worth recalling how gauge invariance is ensured in the Minkowski space. Considering as before a potential  $A_\mu = \text{constant}$ , the time dependence under the integral analogous to Eq. (11) is completely determined by the purely oscillatory factors  $\bar{u}_{\mathbf{p},\lambda} \sim e^{iE_{\mathbf{p}}t}$ ,  $v_{\mathbf{p}',\lambda'} \sim e^{iE_{\mathbf{p}'}t}$ , which implies

$$\mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'} \sim \delta(E_{\mathbf{p}} + E_{\mathbf{p}'}), \quad (19)$$

so that the amplitudes identically vanish. This is rather trivial, so let us recall the case of a general gauge transformation

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda. \quad (20)$$

In these conditions the gauge variations of the amplitudes are

$$\Delta \mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'} = -ie \int d^4x \bar{u}_{\mathbf{p},\lambda} \gamma^\mu v_{\mathbf{p}',\lambda'} (\partial_\mu \Lambda). \quad (21)$$

Assuming that  $\Lambda(x)$  can be Fourier transformed,

$$\Lambda(x) = \int d^4k \hat{\Lambda}(k) e^{ikx}, \quad (22)$$

the Fourier components of Eq. (21) are

$$\sim \delta^4(p + p' - k) \bar{u}_\lambda(\mathbf{p}) (k_\mu \gamma^\mu) v_{\lambda'}(\mathbf{p}'), \quad (23)$$

which again vanish due to the on-shell relations

$$(p_\mu \gamma^\mu - m) u_\lambda(\mathbf{p}) = 0, \quad (p_\mu \gamma^\mu + m) v_\lambda(\mathbf{p}) = 0. \quad (24)$$

This calculation might leave the impression that gauge invariance in dS space is lost due to the absence of on-shell relations like Eqs. (24), as implied by the time-dependent metric. This is however not so: the vanishing of Eq. (23) is actually the Fourier transform of the current-conservation-like relation

$$\partial_\mu (\bar{u} \gamma^\mu v) = 0, \quad (25)$$

which admits the curved spacetime generalization, i.e.

$$\partial_\mu(\sqrt{-g} e_\alpha^\mu \bar{u} \gamma^\alpha v) = 0. \quad (26)$$

In the next section we will reexamine the problem based on Eq. (26), which will allow to precisely identify what goes wrong in dS space.

#### 4. The adiabatic residue

We first recall some known facts from the Minkowski space. A usual way to check the invariance of the amplitudes (11) under the gauge transformations (20) in flat space is to perform an integration by parts and then use Eq. (25). However, the desired property rigorously follows only if one can ignore the surface terms.<sup>e</sup> The contributions from spatial infinity can be ignored with the usual assumption that at infinite distances the physical fields vanish, but this is not allowed for the hypersurfaces at  $t \rightarrow \pm\infty$ . One way to deal with the problem is to decouple the interaction at infinite times, which allows to ignore the surface terms. After this step, the gauge variations of the amplitudes are given by an integral which contains the time derivative of the decoupling factor. In the Minkowski space, the key fact is that for an adiabatic decoupling the purely oscillatory form of the modes always eliminates this term. Let us see how this works in dS space.

We denote the decoupling functions by  $h_\epsilon(t)$ , where  $\epsilon$  is the decoupling parameter. We request as usual

$$\lim_{\epsilon \rightarrow 0} h_\epsilon(t) = 1 \text{ for } t \text{ fixed}, \quad \lim_{t \rightarrow \pm\infty} h_\epsilon(t) = 0 \text{ for } \epsilon > 0 \text{ fixed}. \quad (27)$$

For clarity, we begin with the Minkowski case. Introducing the decoupling function in Eq. (21) and performing the integration by parts one finds

$$\Delta\mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda\lambda'} = ie \int d^4x h'_\epsilon(t) \bar{u}_{\mathbf{p},\lambda} \gamma^0 v_{\mathbf{p}',\lambda'} \Lambda, \quad (28)$$

where the prime denotes derivation with respect to the argument. Gauge invariance requires that in the adiabatic limit  $\epsilon \rightarrow 0$  this quantity must vanish. We make some simplifications at this step. It is evident that the vanishing of Eq. (28) can only result from the integral with respect to  $t$ , i.e.

$$\sim \int_{-\infty}^{\infty} dt h'_\epsilon(t) e^{i(E_{\mathbf{p}} + iE_{\mathbf{p}'})t} \Lambda(t). \quad (29)$$

It is also clear that the vanishing property has nothing to do with the form of  $\Lambda(t)$ , so let us assume that  $\Lambda(t)$  is zero for  $t < 0$  and  $\Lambda(t) = 1$  for  $t \geq 0$ . In these conditions the integral reads

$$R_M(\epsilon) \equiv \int_0^{\infty} dt h'_\epsilon(t) e^{i(E_{\mathbf{p}} + E_{\mathbf{p}'})t}. \quad (30)$$

<sup>e</sup>The potential  $A_\mu$  is not a physical field, so there is no reason to assume that  $\Lambda$  generally vanishes at the boundary of spacetime.

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For example, for an exponential decoupling

$$h_\epsilon(t) = e^{-\epsilon|t|}, \quad R_M(\epsilon) = \frac{-i\epsilon}{E_{\mathbf{p}} + E_{\mathbf{p}'} - i\epsilon}, \quad (31)$$

and as expected the result vanishes for  $\epsilon \rightarrow 0$ . The same test can be applied to any spacetime. We will call integrals similar to Eq. (30) in the limit  $\epsilon \rightarrow 0$  *adiabatic residues*. Gauge invariance of the amplitudes can then be translated by saying that the adiabatic residues must vanish.

We now apply the test to dS space. The first step is to establish the analogue of Eq. (30). We introduce in the general form of the amplitudes (5) the decoupling functions  $h_\epsilon(t)$  and consider the gauge variations implied by the transformations (20). Integrating by parts using Eq. (26) one finds

$$\Delta \mathcal{A}(\mathbf{p}, \mathbf{p}')_{\lambda \lambda'} = ie \int d^4x \sqrt{-g} h'_\epsilon(t) e_{\alpha}^0 \bar{u}_{\mathbf{p}, \lambda} \gamma^\alpha v_{\mathbf{p}', \lambda'} \Lambda. \quad (32)$$

We are interested in the integral with respect to  $t$  in the limit  $\epsilon \rightarrow 0$ . Note that, in contrast to the Minkowski case, there is now an essential past-future asymmetry, as implied by the expansion of space. The important observation is that in the adiabatic limit the derivative of the decoupling function is  $h'_\epsilon(t) \rightarrow 0$  (for  $t$  fixed), which implies that the integral can be non-zero only due to the contributions from infinite times  $t \rightarrow \pm\infty$ . Furthermore, since for  $t \rightarrow -\infty$  the modes become purely oscillatory with respect to  $\eta$ , the picture in this limit is essentially the same with that in the Minkowski space. As a consequence, a non-zero adiabatic residue can only come from the contributions from  $t \rightarrow +\infty$ . This means that we can make the same choice for  $\Lambda(t)$  as in the calculation above.

From now on we consider all quantities as functions of the time  $\eta$ . Notice that the relevant integration interval  $t \in [0, \infty)$  translates into

$$\eta \in [-H^{-1}, 0). \quad (33)$$

Using the explicit form of the modes (8) and (9) one finds that the integral with respect to  $\eta$  in Eq. (32) is a linear combination of integrals of the following form:

$$\sim \int_{-H^{-1}}^0 d\eta h'_\epsilon(\eta) \eta H_{\nu_\pm}^{(2)}(-p\eta) H_{\nu_\mp}^{(2)}(-p'\eta). \quad (34)$$

The distinction between the  $\pm$  cases is inessential, as it amounts to  $p \leftrightarrow p'$ . Hence, the analogue of Eq. (30) can be chosen to be

$$R_{dS}(\epsilon) \equiv p \int_{-H^{-1}}^0 d\eta h'_\epsilon(\eta) \eta H_{\nu_+}^{(2)}(-p\eta) H_{\nu_-}^{(2)}(-p\eta), \quad (35)$$

where for simplicity we set  $p' = p$ . The factor in front of the integral was introduced only for making the expression dimensionless. One can check that in the flat space limit  $H \rightarrow 0$  the integral expressed in terms of the time  $t$  reduces to the Minkowski form (30) with  $E_{\mathbf{p}'} = E_{\mathbf{p}}$  (up to an inessential factor depending on  $p$ ).



It is interesting to explicitly obtain the adiabatic residue implied by Eq. (35) for the exponential decoupling (31). The functions  $h_\epsilon$  in terms of the time  $\eta$  in the interval of interest are

$$h_\epsilon(\eta) = (-H\eta)^{\frac{\epsilon}{H}}, \quad -H\eta \in (0, 1), \quad (36)$$

in which conditions

$$R_{dS}(\epsilon) = \frac{p}{H} \epsilon \int_{-H^{-1}}^0 d\eta (-H\eta)^{\epsilon/H} H_{\nu_+}^{(2)}(-p\eta) H_{\nu_-}^{(2)}(-p\eta). \quad (37)$$

The integral (37) can be evaluated in a straightforward way by expanding the product of Hankel functions as a power series in  $\eta$  using  $(z \equiv -p\eta)$

$$H_\nu^{(2)}(z) = \frac{e^{\nu\pi i} J_\nu(z) - J_{-\nu}(z)}{i \sin \pi \nu}, \quad J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(\nu + n + 1)} \left(\frac{z}{2}\right)^{2n}. \quad (38)$$

The result is rather complicated, but it significantly simplifies for  $\epsilon \rightarrow 0$ . We first observe that the power expansion is of the form

$$H_{\nu_+}^{(2)}(z) H_{\nu_-}^{(2)}(z) = \frac{A_H}{z} + \text{Rest}(z), \quad A_H = -\frac{2}{\pi} \frac{1}{\cosh\left(\frac{\pi m}{H}\right)}, \quad (39)$$

$$\text{Rest}(z) = z^{2ik} B(z) + z^{-2ik} C(z) + zD(z), \quad (40)$$

where the functions  $B$ ,  $C$  and  $D$  are sums over *non-negative* integer powers of  $z$ . Another essential observation is that the factor  $\sim \epsilon$  in front of the integral (37) implies that the limit  $\epsilon \rightarrow 0$  is determined only by the terms in the series which lead to a divergent quantity  $\sim \epsilon^{-1}$ . It is easy to see that such a quantity can be produced only by the term  $\sim \frac{1}{z}$  in Eq. (39). This leaves us with

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} R_{dS}(\epsilon) &= A_H \lim_{\epsilon \rightarrow 0} \epsilon \int_{-H^{-1}}^0 d\eta (-H\eta)^{\epsilon/H-1} \\ &= -A_H \lim_{\epsilon \rightarrow 0} (-\eta H)^{\epsilon/H} \Big|_{\eta=-H^{-1}}^{\eta=0} = A_H. \end{aligned} \quad (41)$$

We have thus obtained that the adiabatic residue in dS space does not vanish. This shows that the dS amplitudes cannot be gauge independent quantities. Note that, as expected, in the limit  $H \rightarrow 0$  we recover the vanishing result from the flat space. A representation of  $R_{dS}(\epsilon)$  defined by Eq. (37) as a function of  $\epsilon$  for different values of  $\frac{H}{m}$  is shown in Fig. 1.

It is remarkable that the same result (41) is obtained irrespective of the form of the decoupling functions. In order to see this it is sufficient to go back to Eq. (35) and repeat the same steps using Eqs. (39) and (40) for an arbitrary  $h_\epsilon(\eta)$ . One similarly finds that for  $\epsilon \rightarrow 0$  the integral reduces to

$$\lim_{\epsilon \rightarrow 0} R_{dS}(\epsilon) = -A_H \lim_{\epsilon \rightarrow 0} \int_{-H^{-1}}^0 d\eta h'_\epsilon(\eta) = -A_H \lim_{\epsilon \rightarrow 0} h_\epsilon(\eta) \Big|_{-H^{-1}}^0 = A_H. \quad (42)$$

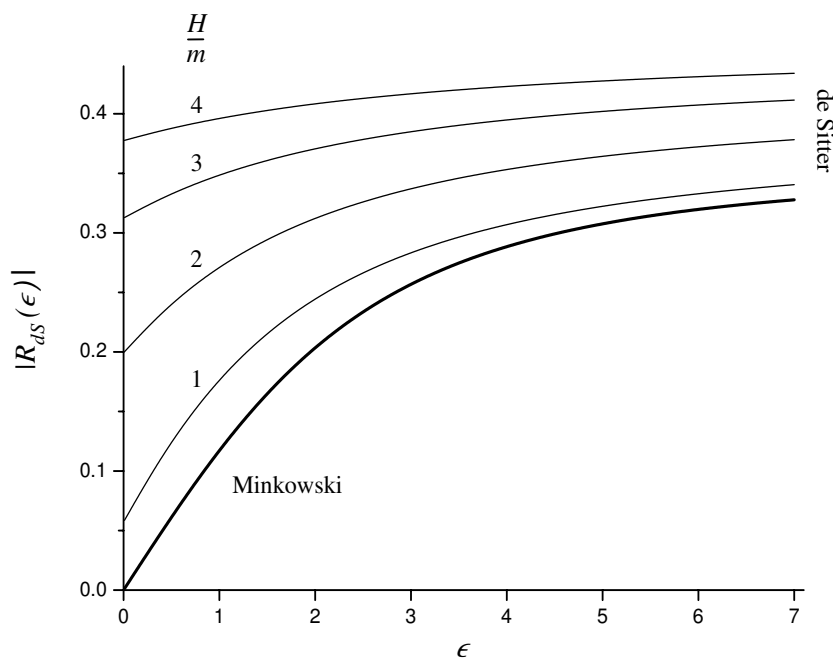


Fig. 1. The absolute value of Eq. (37) represented as a function of  $\epsilon$  for different ratios  $\frac{H}{m}$  (shown near the curves) and  $\frac{p}{m} = 1$ . The curve in bold corresponds to the flat space limit  $H \rightarrow 0$ .

The last identity follows from

$$\lim_{\epsilon \rightarrow 0} h_{\epsilon}(\eta < 0) = 1, \quad \lim_{\eta \rightarrow 0} h_{\epsilon}(\eta) = 0 \mid_{\epsilon > 0}, \quad (43)$$

which are the analogue of Eq. (27) in terms of the time  $\eta$ .

Let us rephrase our calculation. In Minkowski space the vanishing of the adiabatic residue is assured by the fact that the oscillatory form of the modes with respect to time keeps the integral (29) finite, which in the adiabatic limit forces the integral to vanish. This does not happen in dS space, and from the calculation above it is clear that the cause lies in the behavior of the modes for  $\eta \rightarrow 0$ , or equivalently infinite cosmological times  $t \rightarrow \infty$ . It is useful to look at the contributions from this limit in  $R_{dS}(\epsilon)$  in the following way. Note that a purely oscillatory behavior with respect to  $t$  appears in terms of  $\eta$  as

$$e^{\mp i\omega t} \sim \eta^{\pm \frac{i\omega}{H}}. \quad (44)$$

The non-zero result (42) is produced only by the divergent non-oscillatory term  $\sim \frac{1}{\eta}$  in Eq. (39). The rest of the terms (40) are finite oscillatory terms, which are irrelevant in the adiabatic limit. It is crucial to observe that if one replaces  $\frac{1}{\eta}$  in Eq. (39) with a divergent oscillatory term  $\sim \eta^{-1+i\alpha}$  (with  $\alpha$  real) the integral (37)

remains finite for  $\epsilon \rightarrow 0$ , which implies a vanishing adiabatic residue. Hence, in a technical sense, gauge dependence in dS space arises due to the fact that in the integral which defines the gauge variations of the amplitudes the integrand loses its oscillatory behavior at  $\eta \rightarrow 0$ .

We considered in our discussion that the initial and final wave functions in the amplitudes are of the form  $\bar{\psi}_f \psi_i = \bar{u}v$ . It turns out that the situation is practically the same for the combinations  $\bar{u}u$  and  $\bar{v}v$  (corresponding to scattering amplitudes). The only difference in these cases is that the product of Hankel functions in Eq. (35) is replaced by  $H_{\nu_{\pm}}^{(1)}(-p\eta)H_{\nu_{\mp}}^{(2)}(-p'\eta)$ . An identical calculation shows that the expansion in powers of  $z$  has the same form (39), so that the adiabatic residue is non-zero in these cases too. It is interesting that for all possible  $u$ - $v$  combinations the divergent term which is responsible for the non-zero adiabatic residue is a quantity of the form ( $a, b = 1$  or  $2$ )

$$H_{\nu_{\pm}}^{(a)}(-p\eta)H_{\nu_{\mp}}^{(b)}(-p'\eta) \sim \eta^{-(\nu_+ + \nu_-)} = \eta^{-1}, \quad \eta \rightarrow 0. \quad (45)$$

This ‘universal’ dependence on  $\eta$  will be essential when establishing the condition for ensuring the gauge invariance of the amplitudes in Sec. 5.

Let us stress that the oscillatory behavior of the integrand for  $\eta \rightarrow \infty$  in Eq. (34) does not disappear due to the modes themselves, which remain oscillatory in the infinite future. One can easily check that in this limit both types of modes (8) and (9) contain components which oscillate as

$$\eta^{\pm ik} \sim e^{\mp i\eta t}, \quad t \rightarrow \infty. \quad (46)$$

The key fact is that the limit behavior (46) does not depend on the momentum of the modes, which is what leads to the disappearance of the oscillatory behavior for *all* pairs of initial and final momenta  $\mathbf{p}, \mathbf{p}'$ . The fact that the late time frequencies become independent of the comoving momenta can be obviously recognized to be an effect of the arbitrarily large expansion of space at  $t \rightarrow \infty$ . Thus, from a physical point of view, the loss of gauge invariance of the amplitudes is a consequence of this property.

An immediate generalization of the conclusion above is that an identical situation can be expected to occur in a FRW spacetime with the same behavior at late times. It is also evident that if the physical process of interest is sufficiently localized in time so that the arbitrarily large expansion of space at large times can be ignored (the case of usual experiments) the problem will not appear.

For completeness, in Appendix A we included the analogous calculation for the adiabatic residue in scalar QED. The result is very similar to that in Eq. (41).

## 5. Restoring the gauge invariance

We have remarked in a previous work<sup>77</sup> in the context of scalar QED calculation in the same background that the gauge invariance of the amplitudes analogous to Eq. (11) is assured if one restricts to potentials  $A_{\mu}$  which vanish in the infinite future,

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i.e.

$$\lim_{\eta \rightarrow 0} A_\mu(\eta, \mathbf{x}) = 0. \quad (47)$$

Let us show that the same prescription can be applied for spinor QED.

The proof is practically the same with that in Minkowski space, and consists in showing that for the potentials (47) the problematic surface term from  $\eta \rightarrow 0$  which was neglected in the integration by parts (32) can be ignored without introducing the decoupling functions. Considering the general form of the amplitude (5), the surface term of interest is (we continue to use the conformal coordinates)

$$\Delta \mathcal{A}_{i \rightarrow f} = -ie \lim_{\eta \rightarrow 0} \int d^3 \mathbf{x} \sqrt{-g} e_\alpha^0 \bar{\psi}_f \gamma^\alpha \psi_i \Lambda. \quad (48)$$

The only difference with respect to the Minkowski case is that we must be careful about the possible divergences on the integration hypersurface. One finds that for any combination of  $u$ - $v$  modes in the initial and final states the integrand for  $\eta \rightarrow 0$  behaves as (compare with Eq. (45)):

$$\sqrt{-g} e_\alpha^0 \bar{\psi}_f \gamma^\alpha \psi_i \sim \eta H_{\nu_+}^{(a)}(-p\eta) H_{\nu_-}^{(b)}(-p'\eta) \sim \eta^{1-(\nu_++\nu_-)} = 1. \quad (49)$$

The important fact from Eq. (49) is that for the vanishing of the surface term (48) it is sufficient for the function  $\Lambda$  to vanish at  $\eta = 0$ . We observe at this point that the gauge transformations are actually not determined by  $\Lambda$ , but only by the derivatives

$$\Lambda_\mu \equiv \partial_\mu \Lambda. \quad (50)$$

By restricting to the potentials (47) the same condition must be respected by  $\Lambda_\mu$ , and in these conditions one can always consider that  $\Lambda$  has the property mentioned above. This can be easily seen with the redefinition

$$\Lambda(\eta, \mathbf{x}) \rightarrow \Lambda'(\eta, \mathbf{x}) = \int_{(0, \mathbf{x}_0)}^{(\eta, \mathbf{x})} dx^\mu \Lambda_\mu, \quad (51)$$

where  $(0, \mathbf{x}_0)$  is a fixed point on the hypersurface  $\eta = 0$ , and where the line integral runs along an arbitrary curve which connects the two points  $(\partial_\mu \Lambda_\nu = \partial_\nu \Lambda_\mu)$ . By construction, the new partial derivatives are  $\Lambda'_\mu = \Lambda_\mu$ , while on the integration hypersurface  $\Lambda'(0, \mathbf{x}) = 0$ , which ends the proof.

An undesirable feature of Eq. (47) is that it is not a gauge invariant relation. This can be remedied if one strengthens the condition by requesting  $A_\mu$  to smoothly vanish with respect to  $\eta$ , i.e.

$$\lim_{\eta \rightarrow 0} \partial_\eta A_\mu(\eta, \mathbf{x}) = 0. \quad (52)$$

Equations (47) and (52) combined imply  $\lim_{\eta \rightarrow 0} \partial_\mu A_\nu(\eta, \mathbf{x})$ , from which

$$\lim_{\eta \rightarrow 0} F_{\mu\nu}(\eta, \mathbf{x}) = 0, \quad (53)$$

which is in a gauge invariant form. One can easily show that the last condition is essentially equivalent to the first two ones, in the sense that if the electromagnetic tensor respects Eq. (53) one can always choose a potential which respects Eqs. (47) and (52).<sup>77</sup>

We emphasize that all the conditions above are expressed in terms of the conformal coordinates  $(\eta, \mathbf{x})$ . The limits will generally assume a different form in other coordinate systems. For example, if one replaces  $\eta$  with the time  $t$  the temporal component in Eq. (47) becomes (the other components remain unchanged)

$$\lim_{t \rightarrow \infty} (e^{Ht} A_t(t, \mathbf{x})) = 0, \quad (54)$$

which implies a much faster vanishing of  $A_t$  with respect to  $t$ .

We agree that these conditions might appear too restrictive, especially having in mind that in flat space one can deal with even divergent potentials at infinite times. One can take the view that they are the price to be paid for the rather extreme conditions implied by the infinite expansion of space at  $t \rightarrow \infty$ . It remains to be seen with more concrete calculations if our proposal is indeed a solution to the problem.

## 6. Conclusions and further proposals

There exists a lot of work on QED effects in dS space. However, it seems that the question of the gauge invariance has not yet received a detailed investigation. In this paper we brought evidence that in the expanding patch of dS space gauge invariance does not necessarily holds. Our conclusion comes from considering tree-level amplitudes in an external field, with the initial and final particle states defined by the Bunch-Davies modes. A simple illustration is provided by the amplitudes for pure gauges of the form  $A_\mu = \text{constant}$ . One finds that these amplitudes do not generally vanish, which is incompatible with the gauge invariance of the theory.

For a more general analysis, we examined the gauge variations of the amplitudes assuming a decoupling of the interaction at infinite times. The gauge variations are then contained in an integral which contains the time dependent factors of the modes and the time derivative of the decoupling function. In the Minkowski space this integral always vanishes for an adiabatic decoupling, which is assured by the purely oscillatory time dependence of the modes. This does not happen in the expanding dS space, and the cause lies in the fact that the integrand becomes non-oscillatory in the infinite future. However, this does not result from the fact that the modes cease to oscillate in this limit, but due to the special dependence on the oscillatory components in the Dirac current  $\bar{\psi}\gamma^\mu\psi$ , combined with the fact that the late time oscillatory behavior becomes independent on the comoving momenta. Hence, the loss of gauge invariance in dS space can be identified to be an effect of the infinite expansion of space at  $t \rightarrow \infty$ . This suggests that the same problem will appear in more general FRW spaces with the same behavior at large times. The problem, however, will not appear if for the physics of interest the arbitrarily large

expansion of space at late times can be ignored.

We suggested a possible solution for assuring the gauge invariance of the amplitudes, which consists in restricting the calculations to potentials which in conformal coordinates respect  $\lim_{\eta \rightarrow 0} A_\mu = 0$ . The same prescription works both for the scalar and Dirac fields. Such potentials can always be found if  $\lim_{\eta \rightarrow 0} F_{\mu\nu} = 0$ . However, one should be aware that our proof only states the identity between amplitudes in different gauges for which at the infinite future  $\lim_{\eta \rightarrow 0} \Delta A_\mu = 0$ . One could speculate from here that beside the electromagnetic field an extra physical information might be stored in the potential on the conformal boundary of the dS space at  $\eta = 0$ .

Our discussion so far focused on amplitudes in an external classical field. Let us make a few comments on the question of gauge invariance in the full interacting theory with  $A_\mu$  a quantized field. An immediate observation is that in this case the tree amplitudes with a single vertex still have the general form (11), with the only difference that the external potential is replaced by the wave function which corresponds to the photon line. It follows then with the same argument given in Sec. 3 that these amplitudes are also gauge dependent quantities.<sup>f</sup>

Another threat to gauge invariance can come of course from the form of the photon propagator. Given the gauge dependence at the tree level, one can naturally suspect that it will reappear in higher order calculations, and thus be a generic feature of theory. As we remarked in Sec. 1, a possible example is provided by the one loop calculations in Refs. 56, 57. Let us briefly enumerate some options in the face of this possibility.

One view would be to simply accept that gauge dependence is an unavoidable feature of the theory in the expanding dS space, as long as one insists on keeping the background fixed. This could be seen as an unfortunate effect implied by the idealization of an exponential expansion up to infinite times. As suggested by many authors, it is plausible that in a real life scenario quantum backreaction effects will slow down or completely eliminate the expansion, so that in a more complete theory gauge invariance could be recovered. A similar view was adopted in the discussion of the unitarity violation in dS space.<sup>28</sup>

Another possibility is that the problem is more of a technical nature. For example, one could suspect that gauge dependence in the amplitudes considered here is related to the fact that the *out* modes do not describe real physical states. However, this cannot be so, which can be seen in the following way. Assuming distinct sets of *in* and *out* modes, the amplitudes for the transition between the initial vacuum

<sup>f</sup>Amplitudes of this form involving integrals identical or very similar to Eq. (15) appear in Refs. 32–40. As we mention below, a possible way to guarantee the physical relevance of these results is that there exists a preferential gauge to perform the calculations. A natural choice would be the radiation gauge, which is indeed the case in all these papers.

and a final state  $\alpha$  is of the form<sup>75</sup>

$$\mathcal{A}_{0 \rightarrow \alpha} = \langle \alpha \text{ out} | S | 0 \text{ in} \rangle = \sum_{\beta} \langle \alpha \text{ out} | \beta \text{ in} \rangle \langle \beta \text{ in} | S | 0 \text{ in} \rangle, \quad (55)$$

where the scalar products in front of the  $S$ -matrix elements are defined by the Bogolubov coefficients which connect the two sets of modes. It is easy to see that in the case examined here, i.e. first order approximation and  $\alpha$  a particle-antiparticle state, only two terms appear in the sum: one which contains the matrix element  $\langle \alpha \text{ in} | S^{(1)} | 0 \text{ in} \rangle$  and which represents the amplitudes in Sec. 2, and another one which contains the vacuum-to-vacuum amplitude  $\langle 0 \text{ in} | S | 0 \text{ in} \rangle$ . The last amplitude comes multiplied by the factor  $\langle \alpha \text{ out} | 0 \text{ in} \rangle$ , which due to the translational invariance of the initial vacuum is proportional to the delta function  $\delta^3(\mathbf{p} + \mathbf{p}')$ . For an arbitrary external potential  $A_{\mu}(\eta, \mathbf{x})$  such a function will *not* appear in the first term, so that it is impossible for the gauge variations from the two terms to cancel each other.

One can also contemplate the possibility that gauge dependence is a perturbative artifact. It became clear in recent years that infrared divergences in individual Feynman diagrams in dS space can be eliminated by resummation techniques (see e.g. Refs. 78–80), so that something similar could happen for the gauge variations of the amplitudes.

Finally, a simple way out would be that there exists a preferred gauge which ensures physically meaningful results. Our restriction (47) for assuring the gauge invariance of the amplitudes would then suggest to choose a gauge in which the propagators in conformal coordinates vanish when one of the points is at future infinity, i.e.

$$\lim_{\eta \rightarrow 0} \Delta_{\mu\nu'}(\eta, \eta'; \mathbf{x}, \mathbf{x}') = 0, \quad (56)$$

and similarly for the primed point. It is essential in this context to recall that the behavior of the dS photon propagators at large spacetime separations can significantly depend on the choice of gauge.<sup>71</sup> As shown in the cited paper, among the dS invariant propagators in the  $R_{\lambda} = \frac{1}{2}(\nabla_{\mu} A^{\mu})^2$  gauges only the propagator in the Landau gauge  $\lambda \rightarrow \infty$  decreases at large spacetime separations. One can also construct<sup>71</sup> other dS invariant propagators whose transverse (physical) part has the same property, but it is not clear what type of gauge generates them. Unfortunately, one finds<sup>8</sup> that even these well behaved propagators do not respect Eq. (56). One can also check that the same is true for the various propagators used in Refs. 56–66. For the moment, we do not know whether propagators that satisfy the property above exist.

As a final suggestion, it would be interesting to try to obtain the dS photon propagator following the familiar quantization in the radiation gauge in flat space.<sup>81</sup> In this case, one starts with the transversal propagator of the free field, to which

<sup>8</sup>A collection of formulas which allow to easily translate the dS invariant propagators<sup>71</sup> in conformal coordinates can be found in Refs. 56, 68.

one adds the contribution which accounts for the Coulomb interaction, which after eliminating a pure gauge leads to the Lorentz invariant propagator in the Feynman gauge. It would be worthwhile to check if going through the same steps in dS space one can construct a propagator (whether dS invariant or not) which respects Eq. (56).

### Appendix A.

We obtain here the correspondent of the adiabatic residue (41) for scalar QED. Introducing

$$\bar{\nu} = \sqrt{\frac{m^2}{H^2} - \frac{9}{8}}, \quad (\text{A.1})$$

the scalar modes of a definite momentum  $\mathbf{p}$  are given by<sup>75</sup>

$$\varphi_{\mathbf{p}}(\eta, \mathbf{x}) = \frac{\sqrt{\pi/H}}{2(2\pi)^3} \times (-H\eta)^{\frac{3}{2}} e^{\frac{\pi\bar{\nu}}{2}} H_{-i\bar{\nu}}^{(1)}(-p\eta) e^{i\mathbf{p}\cdot\mathbf{x}}. \quad (\text{A.2})$$

We suppose this time that  $\frac{m}{H}$  is sufficiently large so that  $\bar{\nu}$  is real. (For  $\bar{\nu}$  imaginary the modes become non-oscillatory for  $\eta \rightarrow 0$ , which makes them inappropriate for a description in terms of particle states at late times. In addition, in this case the amplitudes given below can diverge.<sup>24</sup>) The amplitudes analogous to Eq. (5) are

$$\mathcal{A}(\mathbf{p}, \mathbf{p}') = -ie \int d^4x \sqrt{-g} g^{\mu\nu} (f_{\mathbf{p}'}^*, i \overset{\leftrightarrow}{\partial}_\mu f_{\mathbf{p}}) A_\nu, \quad (\text{A.3})$$

with the initial and final wave functions equal to  $\varphi_{\mathbf{p}}$  or  $\varphi_{\mathbf{p}'}^*$ . The gauge variations of the amplitudes using a decoupling of the interaction analogous to Eq. (32) are

$$\Delta\mathcal{A}(\mathbf{p}, \mathbf{p}') = ie \int d^4x \sqrt{-g} h'_\epsilon(t) g^{\mu 0} (f_{\mathbf{p}'}^*, i \overset{\leftrightarrow}{\partial}_\mu f_{\mathbf{p}}) \Lambda. \quad (\text{A.4})$$

Let us fix  $f_{\mathbf{p}'}^* = \varphi_{\mathbf{p}'}^*$ . For precision sake, we will now make the distinction between (I) creation-annihilation amplitudes, when  $f_{\mathbf{p}} = \varphi_{\mathbf{p}}^*$ , and (II) scattering amplitudes, when  $f_{\mathbf{p}} = \varphi_{\mathbf{p}}$ . Repeating the construction in Sec. 4 one finds that the quantities analogous to Eq. (35) for the two types of amplitudes can be defined as follows:

$$R_{dS}^{(I)}(\epsilon) \equiv \frac{\epsilon}{H} \int_{-H^{-1}}^0 d\eta (-H\eta)^{\epsilon/H} [H_{+i\bar{\nu}}^{(2)}(-p'\eta) \overset{\leftrightarrow}{\partial}_\eta H_{+i\bar{\nu}}^{(2)}(-p\eta)], \quad (\text{A.5})$$

$$R_{dS}^{(II)}(\epsilon) \equiv \frac{\epsilon}{H} \int_{-H^{-1}}^0 d\eta (-H\eta)^{\epsilon/H} [H_{+i\bar{\nu}}^{(2)}(-p'\eta) \overset{\leftrightarrow}{\partial}_\eta H_{-i\bar{\nu}}^{(1)}(-p\eta)]. \quad (\text{A.6})$$

From the experience of the previous calculation, we know that for the result in the limit  $\epsilon \rightarrow 0$  it is sufficient to keep in the power expansion of the Hankel functions only the terms  $\sim \eta^{-1}$ . For obtaining these terms it is sufficient to use

$$H_\nu^{(1)}(z) \simeq \frac{i}{\sin \pi\nu} \left\{ \frac{e^{-i\pi\nu}}{\Gamma(1+\nu)} \left(\frac{z}{2}\right)^\nu - \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} \right\}, \quad z \rightarrow 0, \quad (\text{A.7})$$



together with  $H_\nu^{(2)}(z) = H_\nu^{(1)}(z)^*$ . A calculation similar to that in Eq. (41) then leads to

$$\lim_{\epsilon \rightarrow 0} R_{dS}^{(I)}(\epsilon) = \frac{2}{i\pi} \frac{e^{-\pi\bar{\nu}}}{\sinh \pi\bar{\nu}} \left\{ \left( \frac{p'}{p} \right)^{-i\bar{\nu}} - \left( \frac{p'}{p} \right)^{i\bar{\nu}} \right\}, \quad (\text{A.8})$$

$$\lim_{\epsilon \rightarrow 0} R_{dS}^{(II)}(\epsilon) = \frac{2}{i\pi} \frac{1}{\sinh \pi\bar{\nu}} \left\{ \left( \frac{p'}{p} \right)^{-i\bar{\nu}} - e^{-2\pi\bar{\nu}} \left( \frac{p'}{p} \right)^{i\bar{\nu}} \right\}. \quad (\text{A.9})$$

The first expression is the analogue of Eq. (41) (actually the latter result is not the full expression due to the simplification in Eq. (34)). Notice the extra factor  $e^{-\pi\bar{\nu}}$  in Eq. (A.8), which implies a larger value for the adiabatic residue (II). This could be translated by saying that the scattering amplitudes are more sensitive to gauge transformations than the creation-annihilation amplitudes. The same can be checked to be true for the Dirac field: one finds that for scattering amplitudes the coefficient  $A_H$  in the power series analogous to Eq. (39) contains an extra factor  $\sim e^{+\pi k}$ , which leads to the same conclusion.

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